MA’RUZA

MURAKKAB FUNKSIYANING HOSILASI. SIRTGA O’TKAZILGAN URINMA TEKISLIK VA NORMAL CHIZIQ TENGLAMALARI. YUQORI TARTIBLI XUSUSIY HOSILA VA TO’LIQ DIFFERENSIAL

Mavzuning rejasi

1. a) Sirtga o’tkazilgan urinma tekislik (uning normali, normalning komponentalari).
   b) Sirtga o’tkazilgan normal chiziq tenglamasi.
2. a) Murakkab funksiyaning ta’rifi va analitik ifodasi.
   b) Murakkab funksiyaning orttirmasi.
   c) Murakkab funksiyaning hosilasi.
3. a) $z = f(x, y)$ funksiya uchun differensiallshning invariantligi.
   b) Yuqori tartibli xususiy hosila va to’liq differensial.

Tayanch so’z va iboralar: $z = f(x, y)$ tenglama bilan ifodalangan sirtning biron nuqtasida sirtga o’tkazilgan urinma tekislik, normal chiziq tenglamasi va yo’nalishi, oshkormas funksiya, yuqori tartibli hosila.

Aytaylik Q tekislik $z = f(x, y)$ tenglama bilan berilgan sirt bo’lsin. $M_0 \in Q$.

$M_0(x_0, y_0, z_0)$. $z - f(x, y) = F(x, y, z) = 0$.

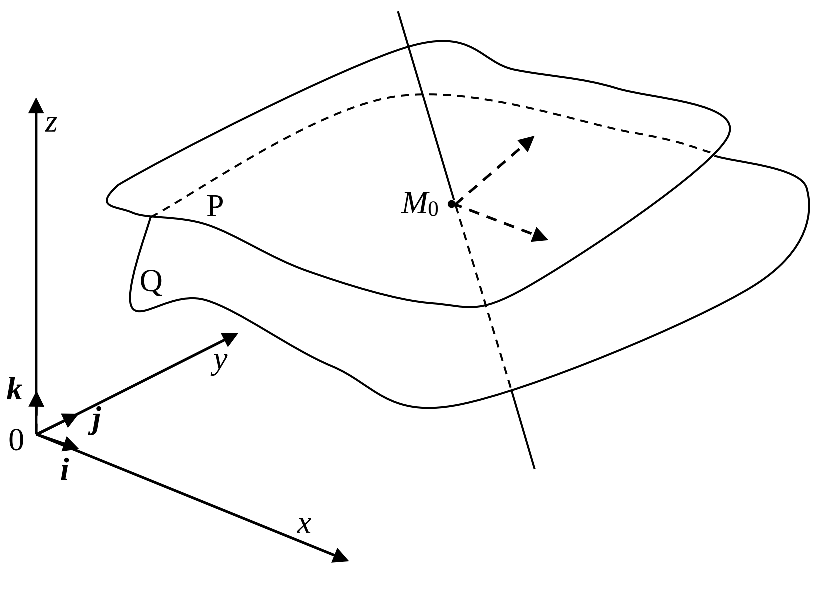
Ta’rif. Egri sirtning biror M nuqtasidan o’tuvchi va shu sirtda yotuvchi egri chiziqlarga urinma joylashgan tekislik sirtga M nuqtada urinma tekislik deb aytiladi.

Bu tekislik $\vec{N} = F'_x \vec{i} + F'_y \vec{j} + F'_z \vec{k}$ vektorga perpendikulyar bo’lgani uchun urinma tekislik

tenglamasini quyidagicha yozaolamiz: $\frac{\partial F}{\partial x}(x - x_0) + \frac{\partial F}{\partial y}(y - y_0) + \frac{\partial F}{\partial z}(z - z_0) = 0$. Agar sirt

tenglamasini oshkor $Z = f(x, y)$ ko’rinishda oladigan bo’lsak, sirt tenglamasi ($\frac{\partial f}{\partial x} = 1$ bo’lgani uchun) quyidagicha yozilgan bo’lardi

$$Z - Z_0 = \frac{\partial f}{\partial x}(x - x_0) + \frac{\partial f}{\partial y}(y - y_0).$$



Ta'rif: Q sirtning $M_0(x_0, y_0, z_0)$ nuqtasi orqali o'tgan va urinma tekislik P ga perpendikulyar bo'lgan chiziq sirtga normal chiziq deb aytiladi.

$M(x, y, z)$ - normal chiziqning ixtiyoriy nuqtasi bo'lsa,

$\overline{H} = \overline{MM_0} = (x - x_0)\overline{i} + (y - y_0)\overline{j} + (z - z_0)\overline{k}$ vector $\overrightarrow{N}$ vektorga parallel bo'ladi. Bundan normal chiziq tenglamasini yoza olamiz

$$\frac{x - x_0}{\frac{\partial F}{\partial x}} = \frac{y - y_0}{\frac{\partial F}{\partial y}} = \frac{z - z_0}{\frac{\partial F}{\partial z}}.$$

Agar tenglama $F(x, y, z) = 0 \Rightarrow Z = f(x, y)$ shaklida yozilgan bo'lsa $\frac{\partial F}{\partial z} = 1$ ekanligini hisobga olib,

normal chiziq tenglamasini $\frac{x - x_0}{-\frac{\partial f}{\partial x}} = \frac{y - y_0}{-\frac{\partial f}{\partial y}} = \frac{z - z_0}{1}$ shaklida yoza olamiz.

Oshkormas funksiya hosilasi

Ta'rif. X va Y ikki faktor bo'lib, o'zaro bog'langan, ammo bu funksional bog'liqlik ularning birontasiga nisbatan yechilmagan bo'lsa, bunday funksional bog'lanish $F(x, y) = 0$ shaklida yoziladi va oshkormas funksiya deyiladi.

Undan hosila olish deganda, bu o'zgaruvchilardan birini (masalan "y")ni x ga bog'liq, ya'ni "x" ni argument deb faraz qilamiz. Bunda $F(x, y) = 0$ ni ikki o'zgaruvchili funksiya deb uning to'liq differensialini olamiz $F_x^{'} dx + F_y^{'} dy = dz = 0$. Bundan $\frac{dy}{dx} = -\frac{F_x^{'}}{F_y^{'}} \Rightarrow y' = -\frac{F_x^{'}}{F_y^{'}}$ ifoda kelib chiqadi.

2. Murakkab funksiyaning hosilasi.

Aytaylik $z = F(u, v)$ (1) funksiya berilgan bo'lib, uning argumentlari u va v-lar o'z isbotida erkli x va y-larning uzluksiz funksiyalari bo'lsin: ya'ni

$$\begin{aligned} u &= \varphi(x, y) \\ v &= \psi(x, y) \end{aligned}. \qquad (2)$$

Bunday holda z funksiya erkli x va y larning murakkab funksiyasi deb aytiladi va quyidagicha yoziladi

$$z = F(\varphi(x, y), v(x, y))$$

(1) va (2) formulalardagi funksiyalar o'z argumentlarining uzluksiz funksiyasi bo'lgani uchun x va y lar Δx va Δy orttirma qabul qilganda u va v ham orttirma qabul qiladi. Natijada z - funksiya ham orttirma qabul qiladi va bunda $\lim\limits_{\Delta x \to 0} \frac{\Delta z}{\Delta x} = \frac{\partial z}{\partial x}$, $\lim\limits_{\Delta x \to 0} \frac{\Delta_x u}{\Delta x} = \frac{\partial u}{\partial x}$,

$\lim\limits_{\Delta x\to 0}\frac{\Delta_x v}{\Delta x}=\frac{\partial v}{\partial x}$ e'tiborga olib, Z funksiyaning to'liq differensialini (u va v bo'yicha) orttirma

qiymat ma'nosida yozib $\Delta Z=\frac{\partial F}{\partial u}\Delta_x u+\frac{\partial F}{\partial v}\Delta_x v+\gamma_1\Delta_x u+\gamma_2\Delta_x v$

Bunda $\lim\limits_{\Delta x\to 0}\gamma_1=0$, $\lim\limits_{\Delta x\to 0}\gamma_2=0$ uni Δx ga bo'lsak,

$$\frac{\Delta Z}{\Delta x}=\frac{\partial F}{\partial u}\frac{\Delta_x u}{\Delta x}+\frac{\partial F}{\partial v}\frac{\Delta_x v}{\Delta x}+\gamma_1\frac{\Delta_x u}{\Delta x}+\gamma_2\frac{\Delta_x v}{\Delta x}$$

ifodani hosil qilamiz. Unda $\Delta x\to 0$ dagi limitini olib,

$$\frac{\partial Z}{\partial x}=\frac{\partial F}{\partial u}\frac{\partial u}{\partial x}+\frac{\partial F}{\partial v}\frac{\partial v}{\partial x} \qquad (1)$$

ni hosil qilamiz. Bu Z funksiyadan x bo'yicha olingan xususiy hosila bo'ladi.

Shuningdek

$$\frac{\partial z}{\partial y}=\frac{\partial F}{\partial u}\frac{\partial u}{\partial y}+\frac{\partial F}{\partial v}\frac{\partial v}{\partial y} \qquad (2)$$

" y " bo'yicha olingan xususiy hosila.

Ma'lumki $$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy \qquad (3)$$

Agar (1) va (2) ni (3) ga qo'ysak

$$dz=\left(\frac{\partial F}{\partial u}\frac{\partial u}{\partial y}+\frac{\partial F}{\partial v}\frac{\partial v}{\partial y}\right)dx+\left(\frac{\partial F}{\partial u}\frac{\partial u}{\partial y}+\frac{\partial F}{\partial v}\frac{\partial v}{\partial y}\right)dy \qquad (4)$$

hosil bo'ladi. (4) ifoda murakkab funksiyaning to'liq differensiali bo'ladi.

Misol. $z=\sin(u+2v)$; $u=x^2+y$, $v=xy$. $dz=?$

$$\frac{\partial z}{\partial x}=\cos(u+2v)\frac{\partial u}{\partial x}+\cos(u+2v)\cdot 2\frac{\partial v}{\partial x}=\cos(u+2v)\cdot 2x+\cos(u+2v)\cdot 2y=$$
$$=2(x+y)\cos(u+2v).$$

$$\frac{\partial z}{\partial y}=\cos(u+2v)\frac{\partial u}{\partial y}+\cos(u+2v)\cdot\frac{\partial v}{\partial y}=\cos(u+2v)\cdot 1+\cos(u+2v)x=$$
$$=(1+x)\cos(u+2v).$$

$$dz=2(x+2y)\cos(u+2v)dx+(1+x)\cos(u+2v)dy.$$

Yuqori tartibli xususiy hosila va to'liq differensial

Soddalik uchun ikki o'zgaruvchili $Z=f(x,y)$ funksiya uchun mavzuni bayon qilamiz (chunki u ko'p argumentli funksiyalarning eng kichik vakili bo'lib, unda bajarilgan barcha differensiallashlar 3, 4 va ko'p argumentlar uchun ham birxildir).

$\frac{\partial Z}{\partial x} = f'_x(x,y)$ bo'lib o'z navbatida x va y larning funksiyalaridir, ya'ni

$f'_x(x,y) = \varphi(x,y)$. Agar $\varphi(x,y)$ - uzluksiz va $\frac{\partial \varphi}{\partial x}$, $\frac{\partial \varphi}{\partial y}$ lar mavjud bo'lsa

$$\frac{\partial \varphi}{\partial x} = \frac{\partial}{\partial x} f'_x(x,y) = \frac{\partial}{\partial x}\frac{\partial}{\partial x} f(x,y) = \frac{\partial^2 f}{\partial x^2} \qquad (5)$$

ifoda $f(x,y)$ funksiyadan olingan II-tartibli xususiy hosila deyiladi.

Shuningdek

$$\frac{\partial \varphi}{\partial y} = \frac{\partial}{\partial y} f'_y(x,y) = \frac{\partial}{\partial y}\frac{\partial f}{\partial x} = \frac{\partial^2 f}{\partial x \partial y} \qquad (6)$$

$f'_y(x,y) = \psi(x,y)$ desak,

$$\frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y} f'_y(x,y) = \frac{\partial}{\partial y}\frac{\partial}{\partial y} f(x,y) = \frac{\partial^2 f}{\partial y^2} \qquad (7)$$

$$\frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x} f'_y(x,y) = \frac{\partial}{\partial x}\frac{\partial}{\partial y} f(x,y) = \frac{\partial^2 f}{\partial x \partial y} \qquad (8)$$

Xususiy hosilalar yana qisqacha f''_{x^2}, f''_{y^2}, f''_{xy}, f''_{yx} - kabi belgilanadi. 3-tartibli Hosilani esa ikkinchi hosilaning hosilasi, shuningdek n-tartibli xususiy hosila $(n-1)$ -tartibli hosilaning hosilasi bo'ladi.

Bu yerda bir asosiy xususiyatni isbotsiz keltiramiz. Agar $Z = f(x,y)$ funksiya va uning xususiy hosilalari f'_x, f'_y, f''_{x^2}, f''_{xy} va f''_{yx} lar $M(x,y)$ nuqta va uning atrofida aniqlangan va uzluksiz bo'lsa, bu nuqtada $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ $\left(f''_{xy} = f''_{yx}\right)$ (9)

bo'ladi. Shu sababli $d^2 Z = \frac{\partial^2 f}{\partial x^2} dx^2 + 2\frac{\partial^2 f}{\partial y \partial x} dxdy + \frac{\partial^2 f}{\partial y^2} dy^2.$ (10)

Misol. $Z = x^2 + y^2 + xy + y$, $Z'_x = 2x + y$, $Z''_{x^2} = 2$, $Z''_{xy} = 1$ (A)

$Z'_y = 2y + x$, $Z''_{y^2} = 2$, $Z''_{yx} = 1$ (B).

Ko'rinib turibdiki (A) va (B) dan $Z''_{xy} = Z''_{yx}$, $d^2 Z = 2dx^2 + 2dxdy + 2dy^2$.